The Neumann-Poincaré operator

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Abstract

Single and double layer potential integral operators have a very long and rich history as a tool to study solutions of boundary value problems. In 1838-1840, Gauss laid the foundations for potential theory, and provided remarkably accurate numerical results based on the single layer potential. Toward the end of the 19th century, Neumann and Poincaré used the double layer potential for the Laplacian, henceforth known as the Neumann-Poincaré (NP) operator, to solve the Dirichlet problem of a domain. Poincaré's method is remarkable for requiring very little boundary regularity; a fact which has only been appreciated very recently.

Meanwhile, in the 20th century, calculating the (essential) spectral radius of the NP operator, for a very large variety of domains and function spaces, became a popular activity. For 2D domains, this is connected to the study of quasiconformal mapping. In this latter context, spectral points of the NP operator are often called Fredholm eigenvalues of the domain. In the 1960-1980s, proving that the NP operator is L^2-bounded for Lipschitz domains was a major motivation for the development of the theory of singular integral operators. More recently, the spectra of NP operators have gained renewed interest, owing to questions from mathematical physics and materials science.

The goal of the course is to give an operator-theoretic introduction to the NP operator. I hope to cover the following topics:

1. The basics of single and double layer potentials
2. Symmetrization theory of operators and basic spectral theory of the NP operator
3. The spectral radius conjecture
4. Spectral theory of the NP operator for planar domains with corners; Mellin convolutions and pseudodifferential operators
5. Spectral theory of the NP operator for domains with singularities in higher dimensions